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Autonomous Observability of Networked Multisatellite Systems

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The focus of this paper is on the development of observability theory and estimation algorithms for multisatellite systems. The results could have applications in space missions that require minimum support from ground control centers and other systems such as the Global Positioning System. The main results consist of 1) the observability of two satellites, either cooperative or noncooperative, using relative measurements only, 2) a computational method for networked multiagent systems to check the observability using their topologies of communication and sensor network, 3) an unscented Kalman filter for the estimation of orbits, positions, and velocities using relative measurements, and 4) simulations on the observability of satellite systems, including a scenario of two satellites and, in another simulation, a networked multisatellite constellation with random communication interruptions.

I. Introduction

OPERATIONS of multisatellite constellations using current technologies, which heavily rely on ground control centers, significantly increase the working load of ground support. Such constellations suffer obvious disadvantages such as the lack of agility and the difficulty of making orbit or formation reconfigurations. In addition, the functionality and the control of the constellation is vulnerable to weather conditions and communication interruptions. Therefore, it is desirable that satellite constellations have the capability of onboard intelligent control that requires less or no ground support. In this paper, the research is aimed at the development of estimation methods for satellite constellations that rely on minimum ground support and Global Positioning System (GPS) signals. More specifically, we prove that it is possible to estimate the orbits, positions, and velocities of space objects using attitude information and relative measurements only. Communication between the objects is not required, although the observabiltiy of networked systems can be improved if such communication is available. In addition, a computational method is developed for networked multiagent systems to check the observability using the topologies of communication and sensor network. In this paper, the results on observability are mathematically proved, and a nonlinear filter is designed using a UKF (unscented Kalman filter). Simulations on the observability of satellite systems are carried out, including a scenario of two satellites and, in another simulation, a networked multisatellite constellation with random communication interruptions.

In Sec. II, we mathematically prove the observability of two satellites using their attitude information and the measurement of relative positions. In Sec. III, we address the observability of networked multiagent systems. In this case, the communication topology and the sensor capabilities are two fundamental elements that compensate to each other. In Sec. IV, a nonlinear filter design using a UKF is outlined. In Sec. V, several simulations are shown to verify and to demonstrate the theory and the algorithms developed.

II. Observability Using Relative Measurements

The observability of networked systems addressed in this paper is derived based on the assumption that some agents are equipped with sensors that are able to measure and estimate the state of itself or other agents. However, measuring and estimating the state of a satellite is highly dependent on the capability of onboard sensors. Although GPS and ground tracking stations provide accurate information about the position, velocity, and orbit of satellites in operation, they suffer the drawbacks of signal loss or delay due to communication interruptions and range limitations. It is desirable to design navigation algorithms that rely on minimum support from another system or agent.

Many results can be found in the literature toward the goal of estimation using information of relative measurements (Doolittle et al. [1], Woffinden and Geller [2], Li and Goodzeit [3], Mukundan et al. [4], Yim et al. [5]). For instance, the estimation of relative orbit elements is addressed in Doolittle et al. [1] using the measurements of relative range from the deputy to the chief and azimuth and elevation angles of the chief as observed from the deputy. In Woffinden and Geller [2], an angles-only navigation filter is developed to determine relative position and attitude between a passive noncooperative target satellite and a maneuvering chaser vehicle. In Li and Goodzeit [3], the observability is examined for a standard geosynchronous spacecraft attitude determination system that uses Earth and sun sensor measurements.

The result developed in this section is an alternative to existing algorithms of state estimation for satellite systems. More specifically, if a satellite (observer) has the information of its absolute attitude and can measure the relative position of another satellite (target), then it is proved that the absolute positions and velocities of both satellites are observable to the observer satellite. Such relative information can be achieved by using multiple cameras or a combination of camera and range sensor. This observability is different from existing results because this method does not require the positions and velocities of either the observer or the target, and the states of both satellites can be estimated using the measurement of relative positions only. The relative dynamics of satellite orbits have been analyzed in the literature by many authors, see Alfriend and Yan [6] and the

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references therein. In this paper, we reveal an underlying relationship between the relative motions and their orbits around the Earth.

A. Definition of Observability

To make the discussion precise, we introduce a concept of observability. It is well known that the definition of linear observability is universal for all linear systems. However, there exist many definitions of observability in the literature of nonlinear control systems. It is a technical question involving several factors, such as the generality of the concept, ease of verification, and the practicality of observer design. In addition, our problem is different from conventional concept of observability in Gauthier and Kupka [7]. Because of the large number of variables involved in multisatellite systems, it could be unnecessarily complicated to derive the observability and to design observers for the entire system of multiple satellites. Therefore, we have to deal with partial observability of networked systems. Furthermore, it is also possible that only a part of the variables are observable, whereas the overall system of systems is unobservable and undetectable. The following definition is appropriate for this study because it takes into consideration partial observability and nonlinear systems, and it avoids the local requirement in some approaches that uses local changes of coordinates (see Gauthier and Kupka [7]). This new definition borrows the idea from the moving horizon type of observers, but it is extended to a much more general case that covers the partial observability of large and complex systems. Consider a general nonlinear control system

$$\begin{aligned} \dot{\xi} &= f(t,\xi,u), \qquad \xi \in \mathbb{R}^n, \qquad u \in \mathbb{R}^m \\ Y &= h(t,\xi,u) \end{aligned} \tag{1}$$

where ξ is the state, u is the control, and Y represents the variable that can be directly measured by sensors. Suppose $z = z(t,\xi,u)$ is a variable to be estimated. In the following, U represents an open and connected set in the time-state-control space $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m$. We assume $u(t)$ is bounded. We also assume it is C^∞ at all but finitely many points in $[t_0, t_1]$. At any time $\bar{t}$, the limits

$$\lim_{t\to\bar{t}^+} u(t), \qquad \lim_{t\to\bar{t}^-} u(t)$$

exist. For the state, we assume $\xi(t)$ is absolutely continuous. If $(\xi(t), u(t))$ satisfies Eq. (1) for all t in $[t_0, t_1]$ except for finitely many points, then $(t, \xi(t), u(t))$ is called a trajectory. In this note, equations involving $u(t)$ always mean "equal almost everywhere," and the notation is "a.e. in $[t_0, t_1]$."

Definition 1: The function $z = z(t,\xi,u)$ is said to be observable in U if for any two trajectories $(t, \xi^i(t), u^i(t))$, $i = 1, 2$ in U defined on a same interval $[t_0, t_1]$, the equality

$$h(\xi^1(t), u^1(t)) = h(\xi^2(t), u^2(t)), \quad \text{a.e. in } [t_0, t_1]$$

implies

$$z(t, \xi^1(t), u^1(t)) = z(t, \xi^2(t), u^2(t))$$

a.e. in $[t_0, t_1]$.

Suppose for any trajectory $(t, \xi(t), u(t))$ in U there always exists an open set $U_1 \subset U$ so that $(t, \xi(t), u(t))$ is contained in U_1 and $z(t,\xi,u)$ is observable in U_1. Then, $z = z(t,\xi,u)$ is said to be locally observable in U.

For a linear system, the observability of all state variables, that is, the case $z(t,\xi,u) = \xi$, implies the classical definition of observability. In this case, the observability matrix has full rank. For nonlinear systems, all state variables in a uniformly observable system must be observable under this definition. If a variable is observable, it can be estimated by Kalman-type filters , or moving-horizon-type observers (Runge–Kutta–Newton observer in Kang [8] or pseudospectral observer in Gong et al. [9]). For satellite systems, this observability is realized using an unscented Kalman filter, which is addressed in Sec. IV. It is possible to verify the observability of variables by checking rank conditions. In Kang and Barbot [10], sufficient conditions for this definition of observability are proved. The following Lemmas, introduced without proof, are some useful special cases of the necessary conditions from Kang and Barbot [10].

Lemma 1: Consider a system without control

$$\dot{\xi} = f(t,\xi), \qquad \xi \in \mathbb{R}^n \qquad Y = h(t,\xi) \tag{2}$$

Let $U \subset \mathbb{R} \times \mathbb{R}^n$ be an open set. Suppose there exists a function $g(\cdot)$ and an integer $k > 0$ so that

$$z = z(t,\xi) = g(t, Y, DY, \ldots, D^{k-1}Y)$$

for all $(t,\xi) \in U$, where D is the differentiation operator. Then $z(t,\xi)$ is observable.

Lemma 2 Consider the system defined by Eq. (2). Let $U \subset \mathbb{R} \times \mathbb{R}^n$ be an open set. Suppose $z(\xi) \in \mathbb{R}$ is a variable to be estimated. Let D be the differentiation operator. Consider

$$V = (Y^T, DY^T, \ldots, D^{l-1}Y^T)^T \tag{3}$$

for some $l > 0$. If $\text{rank}(\frac{\partial V}{\partial \xi}) = n$ for $(t,\xi) \in U$, then $z = z(\xi)$ is locally observable in U.

B. Observability with the Measurement of Relative Position

Consider two satellites, sat-1 and sat-2. Let $\mathbf{r}$ be the vector from the center of the Earth to a satellite. Then the dynamics of the satellite satisfies

$$\ddot{\mathbf{r}}(t) = \frac{\mu}{r^3}\mathbf{r} + \mathbf{a}(\mathbf{r}, t)$$

where μ is a constant, $r = |\mathbf{r}(t)|$, $\mathbf{a}(\mathbf{r}, t)$ is the orbital perturbation such as the J_2 effect. In this paper, perturbations are not taken into consideration. In fact, a perturbation with model usually is auxiliary to making orbits more distinguishable. Nevertheless, this is still an open problem for future research. In an inertial frame $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$ with the origin fixed at the center of the Earth, the state variables of the ith satellite is denoted by

$$x_i, \quad y_i, \quad z_i, \quad v_{x_i} = \dot{x}_i, \quad v_{y_i} = \dot{y}_i, \quad v_{z_i} = \dot{z}_i, \quad r_i = |\mathbf{r}_i|$$

The dynamic model of a satellite system is defined as follows:

$$\begin{aligned} \dot{x}_i &= v_{x_i} \\ \dot{y}_i &= v_{y_i} \\ \dot{z}_i &= v_{z_i} \\ \dot{v}_{x_i} &= -\frac{\mu}{r_i^3} x_i \\ \dot{v}_{y_i} &= -\frac{\mu}{r_i^3} y_i \\ \dot{v}_{z_i} &= -\frac{\mu}{r_i^3} z_i \end{aligned} \tag{4}$$

where $i = 1, 2$. In this formulation, the system is defined by the ordinary differential equation of orbital dynamics. Let $\tilde{\mathbf{r}}$ be the vector from sat-1 to sat-2. Assume this vector can be measured by sat-1 using a laser sensor or through other means. We do not assume any information about the absolute position and velocity relative to the Earth. Assume the relative position vector $\tilde{\mathbf{r}}$ is represented in a nonrotating frame $\{\tilde{\mathbf{E}}_1, \tilde{\mathbf{E}}_2, \tilde{\mathbf{E}}_3\}$ with the origin fixed on sat-1. For the simplicity of the system model, it is required that the directions of this frame coincide with those of $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$. This assumption implies that sat-1 has its attitude information. The coordinates of $\tilde{\mathbf{r}}$ are denoted by

$$[\tilde{x} \quad \tilde{y} \quad \tilde{z}]^T$$

In the following, we analyze the observability of the relative velocity, the absolute position, and the absolute velocity, of both sat-1 and sat-2 based on the measurement of $\tilde{\mathbf{r}}$. The goal of this section is to mathematically prove that it is possible to estimate the absolute positions and velocities of both sat-1 and sat-2 provided that one satellite is able to measure the relative position of the other and they have different orbital radius.

For sat-1 and sat-2, we know $\tilde{\mathbf{r}} = \mathbf{r}_2 - \mathbf{r}_1$. To model the observability problem, we define

$$\dot{\tilde{x}} = \tilde{v}_{\tilde{x}}, \qquad \dot{\tilde{y}} = \tilde{v}_{\tilde{y}}, \qquad \dot{\tilde{z}} = \tilde{v}_{\tilde{z}}$$
$$\dot{x}_i = v_{x_i}, \qquad \dot{y}_i = v_{y_i}, \qquad \dot{z}_i = v_{z_i}$$
$$r_i = |\mathbf{r}_i|$$

To simplify the notation, we omit the subindex 1 for sat-1. For instance, x, y, and z represent x_1, y_1, and z_1. Because of the nonlinearity in the orbital dynamics, the relative dynamics is coupled with the orbital dynamics of the satellites. This nonlinear relationship is assistive to the estimation of the absolute positions and absolute velocities of the satellites

$$\dot{\tilde{x}} = \tilde{v}_{\tilde{x}}, \qquad \dot{\tilde{y}} = \tilde{v}_{\tilde{y}}, \qquad \dot{\tilde{z}} = \tilde{v}_{\tilde{z}}$$
$$\dot{\tilde{v}}_{\tilde{x}} = -\frac{\mu}{r_2^3}(x + \tilde{x}) + \frac{\mu}{r^3}x, \qquad \dot{\tilde{v}}_{\tilde{y}} = -\frac{\mu}{r_2^3}(y + \tilde{y}) + \frac{\mu}{r^3}y$$
$$\dot{\tilde{v}}_{\tilde{z}} = -\frac{\mu}{r_2^3}(z + \tilde{z}) + \frac{\mu}{r^3}z$$
$$\dot{x} = v_x, \qquad \dot{y} = v_y, \qquad \dot{z} = v_z$$
$$\dot{v}_x = -\frac{\mu}{r^3}x, \qquad \dot{v}_y = -\frac{\mu}{r^3}y, \qquad \dot{v}_z = -\frac{\mu}{r^3}z$$
$$\text{Output: } Y = [\tilde{x} \quad \tilde{y} \quad \tilde{z}]^T \tag{5}$$

Given the output Y, we would like to know the observability of all the other state variables. For the purpose of applying the Lemmas in Sec. II.A, ξ in Eq. (2) corresponds to the states variables in Eq. (5), that is, $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{v}_{\tilde{x}}, \tilde{v}_{\tilde{y}}, \tilde{v}_{\tilde{z}}, x, y, z, v_x, v_y$, and v_z; $f(t, \xi)$ is the right side of the differential equations in Eq. (5), and $h(t, \xi)$ represents $[\tilde{x} \quad \tilde{y} \quad \tilde{z}]^T$. The variables to be estimated $z(t, \xi)$ are $\tilde{v}_{\tilde{x}}, \tilde{v}_{\tilde{y}}, \tilde{v}_{\tilde{z}}, x, y, z$, v_x, v_y, and v_z. We denote the differentiation operator by D, then

$$Y = [\tilde{x} \quad \tilde{y} \quad \tilde{z}]^T \qquad DY = [\tilde{v}_{\tilde{x}} \quad \tilde{v}_{\tilde{y}} \quad \tilde{v}_{\tilde{z}}]^T$$
$$D^2Y = \begin{bmatrix} -\frac{\mu}{r_2^3}(x + \tilde{x}) + \frac{\mu}{r^3}x \\ -\frac{\mu}{r_2^3}(y + \tilde{y}) + \frac{\mu}{r^3}y \\ -\frac{\mu}{r_2^3}(z + \tilde{z}) + \frac{\mu}{r^3}z \end{bmatrix}$$
$$D^3Y = \frac{3\mu}{r_2^5}((x + \tilde{x})(v_x + \tilde{v}_{\tilde{x}}) + (y + \tilde{y})(v_y + \tilde{v}_{\tilde{y}}) + (z + \tilde{z})(v_z + \tilde{v}_{\tilde{z}}))\begin{bmatrix} x + \tilde{x} \\ y + \tilde{y} \\ z + \tilde{z} \end{bmatrix} - \frac{\mu}{r_2^3}\begin{bmatrix} v_x + \tilde{v}_{\tilde{x}} \\ v_y + \tilde{v}_{\tilde{y}} \\ v_z + \tilde{v}_{\tilde{z}} \end{bmatrix} - \frac{3\mu}{r^5}(xv_x + yv_y + zv_z)\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \frac{\mu}{r^3}\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

The system has 12 state variables. We stop at the third-order derivative of Y because the Jacobian of $[Y^T \quad DY^T \quad D^2Y^T \quad D^3Y^T]^T$ is a square matrix. If it has full rank, then the observability can be proved. This Jacobian matrix is defined by

$$J = \frac{\partial}{\partial(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{v}_{\tilde{x}}, \tilde{v}_{\tilde{y}}, \tilde{v}_{\tilde{z}}, x, y, z, v_x.v_y.v_z)}\begin{bmatrix} Y \\ DY \\ D^2Y \\ D^3Y \end{bmatrix} \tag{6}$$

The observability of the state variables is closely related to the rank of the Jacobian matrix. It is observed that the Jacobian matrix has lower block triangular structure, with 3×3 blocks on the diagonal. In fact, the submatrix in J has the following properties

$$\frac{\partial}{\partial(\tilde{x}, \tilde{y}, \tilde{z})}Y = I, \qquad \text{other partial derivatives are zeros}$$
$$\frac{\partial}{\partial(\tilde{v}_{\tilde{x}}, \tilde{v}_{\tilde{y}}, \tilde{v}_{\tilde{z}})}DY = I, \qquad \text{other partial derivatives are zeros}$$
$$\frac{\partial}{\partial(x, y, z)}D^2Y = Q, \qquad \frac{\partial}{\partial(v_x.v_y.v_z)}D^2Y = 0$$
$$\frac{\partial}{\partial(v_x, v_y, v_z)}D^3Y = Q$$

where I is the identity matrix and Q is the following 3×3 matrix:

$$Q = \frac{3\mu}{r_2^5}\begin{bmatrix} x + \tilde{x} \\ y + \tilde{y} \\ z + \tilde{z} \end{bmatrix}[x + \tilde{x} \quad y + \tilde{y} \quad z + \tilde{z}] - \frac{\mu}{r_2^3}I - \frac{3\mu}{r^5}\begin{bmatrix} x \\ y \\ z \end{bmatrix}[x \quad y \quad z] + \frac{\mu}{r^3}I$$

Therefore, the Jacobian matrix has the following triangular structure:

$$J = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ J_{31} & J_{32} & Q & 0 \\ J_{41} & J_{42} & J_{43} & Q \end{bmatrix} \tag{7}$$

where J_{ij} are submatrices that do not change the rank of the Jacobian (in fact $J_{32} = 0$). To find the rank of the Jacobian matrix, we must find out the rank of Q. It is important to point out that the rank of J is intrinsic, that is, it is invariant under change of coordinates. If one can prove Q has full rank at an arbitrary time moment, then Q has full rank all time. Now, let us focus on an arbitrary but frozen time moment. Because the rank of Q does not change under change of coordinates, we can use an appropriately selected coordinate frame to simplify Q. For this purpose, we can use a change of coordinates so that the point $\mathbf{r} = x\mathbf{E}_1 + y\mathbf{E}_2 + z\mathbf{E}_3$ has the coordinates $(x, 0, 0)$ at this time moment. For the vector $\tilde{\mathbf{r}} = \tilde{x}\tilde{\mathbf{E}}_1 + \tilde{y}\tilde{\mathbf{E}}_2 + \tilde{z}\tilde{\mathbf{E}}_3$, we can turn the axis $\tilde{\mathbf{E}}_2$ and $\tilde{\mathbf{E}}_3$ around $\tilde{\mathbf{E}}_1$ so that $z + \tilde{z} = 0$, that is, the center of the Earth and the two satellites form $\tilde{\mathbf{E}}_1\tilde{\mathbf{E}}_2$-plane. Under this coordinate system, Q has the following form:

$$Q = \frac{3\mu}{r_2^5}\begin{bmatrix} (x + \tilde{x})^2 & (x + \tilde{x})(y + \tilde{y}) & 0 \\ (x + \tilde{x})(y + \tilde{y}) & (y + \tilde{y})^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{\mu}{r_2^3}I - \frac{3\mu}{r^5}\begin{bmatrix} r^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\mu}{r^3}I$$

In fact, $y = 0$ in Q. But its value does not affect the following argument. If $r = r_2$, then $\text{rank}(Q) = 2$. If $r \neq r_2$, define $\alpha = -\frac{\mu}{r_2^3} + \frac{\mu}{r^3}$, then

$$Q = \begin{bmatrix} \frac{3\mu}{r_2^5}(x + \tilde{x})^2 + \alpha - \frac{3\mu}{r^3} & \frac{3\mu}{r_2^5}(x + \tilde{x})(y + \tilde{y}) & 0 \\ \frac{3\mu}{r_2^5}(x + \tilde{x})(y + \tilde{y}) & \frac{3\mu}{r_2^5}(y + \tilde{y})^2 + \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

The matrix Q is not full rank if and only if the determinant is zero, which is equivalent to say

$$\alpha\frac{3\mu}{r_2^5}(x+\tilde{x})^2+\alpha\frac{3\mu}{r_2^5}(y+\tilde{y})^2+\alpha^2-\frac{9\mu^2}{r^3r_2^5}(y+\tilde{y})^2-\alpha\frac{3\mu}{r^3}=0$$
$$\Rightarrow\frac{9\mu^2}{r^3r_2^5}(y+\tilde{y})^2=3\alpha\left(\frac{\mu}{r_2^3}-\frac{\mu}{r^3}\right)+\alpha^2$$
$$\Rightarrow\frac{9\mu^2}{r^3r_2^5}(y+\tilde{y})^2=-2\alpha^2$$

This is impossible because $-\alpha^2<0$ and $\frac{9\mu^2}{r^3r_2^5}(y+\tilde{y})^2\geq 0$. Therefore, $\text{rank}(Q)=3$ if and only if $r\neq r_2$.

Define

$$V=[\,Y^T \quad DY^T \quad D^2Y^T \quad D^3Y^T\,]^T$$

Because of Eq. (7), the associated Jacobi matrix J defined by Eq. (6) is block-lower triangular. All blocks on the diagonal equal the identity matrix except for Q. So, J has full rank if and only if Q has full rank. This is equivalent to $r\neq r_2$. From Lemma 2, the observability of state variables, including the position and velocity, is implied by $r\neq r_2$. In addition, Lemma 1 entails that the velocities $\tilde{v}_{\tilde{x}}$, $\tilde{v}_{\tilde{y}}$, and $\tilde{v}_{\tilde{z}}$ are always observable, irrelevant to the orbits.

Theorem 3: All state variables in the system in Eq. (5) are locally observable if $r\neq r_2$ on the trajectory. The variables $\tilde{v}_{\tilde{x}}$, $\tilde{v}_{\tilde{y}}$, and $\tilde{v}_{\tilde{z}}$ are always observable.

This theorem implies that it is possible to estimate the absolute positions and velocities of satellites with relative measurements and the attitude information, provided that the target satellite has a different orbital radius. This observability is realized in Sec. IV by an unscented Kalman filter.

III. Observability of Networked Systems

In this section we address the relationship between the observability and the topologies of communication and sensor networks. Although we focus on satellite systems in this paper, the results in this section are applicable to general networked systems. Consider a multiagent system, such as a multisatellite constellation, with a partial communication and sensor network that is dynamically changing during the operation. The observability of the networked system is contingent on the capability of the onboard communication and sensor payload of the agents. In addition, the communication topology and the sensor capability are two fundamental elements that compensate to each other in the determination of the state of each agent. For instance, an agent with greater sensor capability requires less communication in the determination of the states of other agents. On the other hand, an agent without onboard sensors must communicate with other cooperative agents that have the sensor information. In this section, a computationally checkable criteria of observability is developed based on the topologies of the communication and sensor networks.

The communication topology of multiple agent systems is represented by a graph (Fig. 1). Suppose k is the total number of agents. Then this graph can be represented by the communication matrix

$$C=\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1k}\\ c_{21} & c_{22} & \cdots & c_{2k}\\ \vdots & \vdots & \vdots & \vdots\\ c_{k1} & c_{k2} & \cdots & c_{kk}\end{bmatrix}$$

where $c_{ij}=0$ if agent-j cannot receive information from agent-i; $c_{ij}=1$ if agent-j is able to receive the information sent by agent-i. In the case of two way communications, it becomes a symmetric matrix of 0s and 1s. We define $c_{ii}=0$.

As shown in Theorem 3, a satellite is able to estimate the state of another satellite using relative measurements only. However, the capability of making such measurements depends on the capacity of the onboard sensor system and the distance between the satellites. It is a rare situation in which a multiagent system is completely observable all the time by direct sensor measurements between all agents. Therefore, the sensors distributed among the agents form a topology, which can be represented by a directed graph (Fig. 2). The nodes in the graph represent the agents. A directed edge from agent-i to agent-j implies that agent-i is able to measure and estimate the state of agent-j. The graph of a sensor system is algebraically represented by the matrix

$$S=\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1k}\\ s_{21} & s_{22} & \cdots & s_{2k}\\ \vdots & \vdots & \vdots & \vdots\\ s_{k1} & s_{k2} & \cdots & s_{kk}\end{bmatrix}$$

where $s_{ij}=0$ if there is no edge from agent-i pointing to agent-j, that is, agent-i is not able to measure and estimate the state of agent-j; $s_{ij}=1$ if there is an edge from agent-i pointing to agent-j, that is, agent-i is able to measure and estimate the state of agent-j.

The matrix S has a special property if the observability in Theorem 3 is used. In this case, agent-i is able to estimate the state of itself if it can make the relative measurement to any other agent. Therefore, $s_{ii}=1$ if any other component in the ith row of S is nonzero. This property is used to generate S in the simulations in Sec. V.B.

The communication network is able to significantly enhance the capability of the sensors distributed in scattered locations. On the other hand, the onboard sensors make it possible to detect the state of other agents with less communication requirement. Among the claims in the next property we would like to emphasize that, even if both the communication and sensor graphs are disconnected, it is still possible for an agent to receive or estimate the states of all other agents provided that the sensor topology is designed in conjunction with the communication topology. In the following, the graph of the communication topology is denoted by $\mathcal{C}$ and the graph of the sensor topology is denoted by $\mathcal{S}$.

Proposition 1: Agent-i is able to estimate the state of agent-j if one of the following conditions holds: 1) There exists an edge from agent-i pointing to agent-j in the graph $\mathcal{S}$; 2) There exists an agent-p, a path in the graph $\mathcal{C}$ connecting agent-i and agent-p, and an edge from agent-p pointing to agent-j in the graph $\mathcal{S}$.

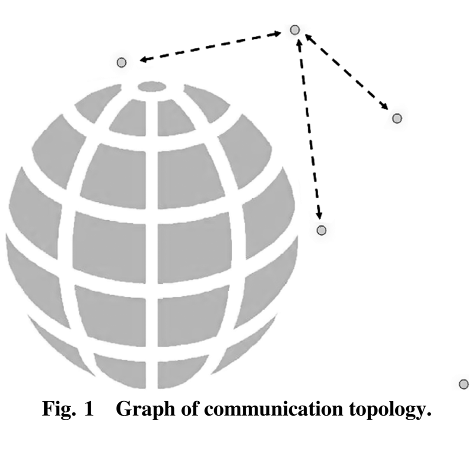

Fig. 1 Graph of communication topology.

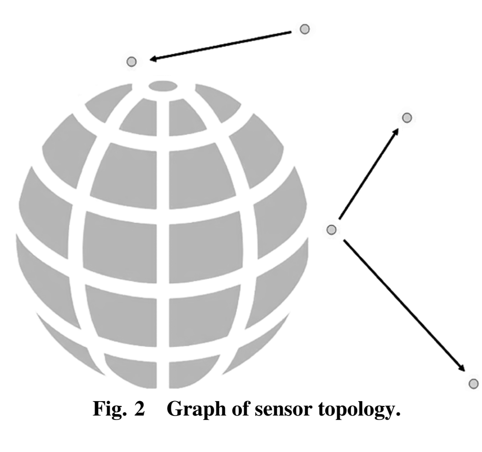

Fig. 2 Graph of sensor topology.

The proof of this proposition is straightforward. It is equivalent to the fact that agent-i is able to estimate the state of agent-j if it has the sensor to directly measure agent-j or it has the communication to a third satellite that has the sensor to measure agent-j.

For the purpose of autonomous planning and outer-loop control, it is important to develop numerically checkable conditions for the observability of a satellite constellation. This is made possible by using the algebraic representations of the communication and the sensor topologies. Using the matrices S and C, Proposition 1 is equivalent to the following fact: agent-i is able to estimate the state of agent-j if it holds that either 1) $s_{ij} = 1$, or 2) there exists integers $i_0, \ldots, i_l$ and p so that

$$i_0 = i; \qquad i_l = p \qquad c_{i_0 i_1} c_{i_1 i_2} c_{i_2 i_3} \cdots c_{i_{l-1} i_l} s_{pj} = 1$$

Note that $c_{i_0 i_1} c_{i_1 i_2} c_{i_2 i_3} \cdots c_{i_{l-1} i_l} s_{pj}$ is a term in the (i, j) component of the matrix $C^l S$. From graph theory (see West [11]), it is known that Proposition 1 is equivalent to the following result.

Proposition 2: Agent-i is able to estimate the state of agent-j if there exists an integer $l \geq 0$ so that the (i, j) component in $C^l S$ is nonzero.

The matrix $C^l S$ in Proposition 2 carries more information then just the observability. In fact, the elements in this matrix represent the number of communication and sensing paths that link the corresponding agents. A larger element in $C^l S$ implies higher reliability of the communication and sensor network due to its redundancy. The following proposition is a conclusion from the properties of adjacency matrices in graph theory [11].

Proposition 3: Let d_{ij}^l denote the ijth element of $C^l S$. If $d_{ij}^l > 0$, then d_{ij}^l equals the number of communication paths of length l that are available for agent-i to inquire the state information of agent-j.

In networked dynamical systems, such as multisatellite constellations, the communication and sensor topologies are not fixed during operations. Propositions 2 and 3 facilitate a computationally verifiable algorithm that can be used to keep tracking in real time the observability and the communication redundancy between agents, which is a piece of essential information for the control of networked systems.

The communication paths in Proposition 3 may include loops and intersections, that is, in the terminology of graph theory they are called walks. However, to count the number of different communication paths without loops and intersections is NP-hard (Jungnickel [12]) and it is beyond the scope of this paper.

IV. Nonlinear Filter Design

The results in Sec. II.B guarantee the observability only. It still remains a problem to estimate the state of satellites using relative measurements. In this section, we develop an estimation method that is facilitated by the UKF (see Julier and Uhlmann [13]). We assume the only measurement that can be used in the estimation is the relative position between the satellites, that is, the output function h is defined by

$$h(\cdot) = [\, x_1 - x_2 \quad y_1 - y_2 \quad z_1 - z_2 \,]^T$$

Consider the following satellite dynamics as a part of Eq. (5):

$$\dot{x} = v_x, \qquad \dot{y} = v_y, \qquad \dot{z} = v_z$$
$$\dot{v}_x = -\frac{\mu}{r^3} x, \qquad \dot{v}_y = -\frac{\mu}{r^3} y, \qquad \dot{v}_z = -\frac{\mu}{r^3} z$$

where $r = \sqrt{x^2 + y^2 + z^2}$ and μ is the Earth's gravitational constant. A difficulty in the numerical experimentation of satellite systems is the extremely large range of scales of the variables and parameters in the model. We found that the following rescaling of the time and state variables significantly improves the accuracy of our numerical integration:

$$t = \sqrt{\frac{r_0^3}{\mu}} \tau; \qquad x = r_0 \bar{x}; \qquad y = r_0 \bar{y}; \qquad z = r_0 \bar{z}$$
$$v_x = \sqrt{\frac{\mu}{r_0}} \bar{v}_x; \qquad v_y = \sqrt{\frac{\mu}{r_0}} \bar{v}_y; \qquad v_z = \sqrt{\frac{\mu}{r_0}} \bar{v}_z$$

where r_0 can be any constant. In the simulations, r_0 equals the radius of the Earth. After the rescaling, the dynamics is transformed into

$$\dot{\bar{x}} = \bar{v}_x, \qquad \dot{\bar{y}} = \bar{v}_y, \qquad \dot{\bar{z}} = \bar{v}_z$$
$$\dot{\bar{v}}_x = -\frac{1}{\bar{r}^3} \bar{x}, \qquad \dot{\bar{v}}_y = -\frac{1}{\bar{r}^3} \bar{y}, \qquad \dot{\bar{v}}_z = -\frac{1}{\bar{r}^3} \bar{z}$$

where $\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}$.

The UKF is "founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation" [13]. In the following, the state variables of Eq. (5) are denoted by ξ. The UKF assumes that at every sampling instant, the state ξ is always a normally distributed variable. The mean and covariance information of this random variable can be stored in a set of specially selected points called sigma points. These sigma points are easily obtained from the method described in Julier and Uhlmann [13]. It can be shown that the nonlinear transformation of the sigma points preserves statistics up to the second order in a Taylor series expansion. Based on this fact, a prediction of the state and the covariance matrices in the filter algorithm can be carried out as follows:

1) From the previous estimate of the state $\hat{\xi}_{n-1}$ and the covariance matrix $\hat{P}_{n-1}^{\xi\xi}$, calculate a set of sigma points,

2) Propagate all the sigma points through the nonlinear dynamic and the output equations. Also propagate $\hat{\xi}_{n-1}$ to obtain $\tilde{\xi}_n$.

3) Predict the covariance matrices by computing $\tilde{P}_n^{\xi\xi}$, $\tilde{P}_n^{YY}$, and $\tilde{P}_n^{\xi Y}$ using a statistical procedure.

4) Use the predicted value of $\tilde{\xi}_n$, $\tilde{P}_n^{\xi\xi}$, $\tilde{P}_n^{YY}$, and $\tilde{P}_n^{\xi Y}$ to update the estimations as follows:

$$\hat{\xi}_n = \tilde{\xi}_n + K(Y_n - \tilde{Y}_n)$$
$$K = \tilde{P}_n^{\xi Y} [\tilde{P}_n^{YY}]^{-1}$$
$$\hat{P}_n^{\xi\xi} = \tilde{P}_n^{\xi\xi} - K \tilde{P}_n^{YY} K^T$$

The detailed formulae for the UKF can be founded in Julier and Uhlmann [13]. In the next section, we apply the UKF filter to the scaled model introduced at the beginning of this section to estimate the absolute positions, the absolute velocities, and the orbits of both satellites using relative measurements. Then, the computational results are transformed back to its original scale for analysis and presentation.

V. Simulations

In this section, simulations are carried out to verify and to illustrate the results obtained in previous sections. We first simulate the state estimation of two satellites. Then, a simulation is carried out for the observability of a networked multisatellite constellation.

A. State and Orbit Estimation of Two Satellites

In the simulations, we consider two satellites flying in two different orbits. The orbital parameters are defined in Table 1. Transferring the orbital parameters to state vectors we get the initial conditions for the satellites, shown in Table 2. With this set of initial conditions, the distance between the two satellites, that is

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

is approximately between 100 km and 250 km (see Fig. 3a). The radial difference between the two selected orbits is defined by

Table 1 The orbital parameters used for the simulations

	Satellite 1	Satellite 2
Semimajor axis	10,000 km	10,000 km
Eccentricity	0.1	0.11
Inclination	0	0.0166668°
Periapsis	0	0
Ascending node	0	0
Mean anomaly	0	0

Table 2 Initial satellite conditions

	Satellite 1	Satellite 2
(x, y, z)	(9000, 0, 0) km	(8900, 0, 0) km
(v_x, v_y, v_z)	(0, 6.980, 0) km/s	(0, 7.050751, 0.002051) km/s
$(\bar{x}, \bar{y}, \bar{z})$	(1, 0, 0)	(0.9889, 0, 0)
$(\bar{v}_x, \bar{v}_y, \bar{v}_z)$	(0, 1.0488, 0)	(0, 1.0595, 0.0003)

$$e_r := r_1 - r_2, \quad \text{where } r_i := \sqrt{x_i^2 + y_i^2 + z_i^2}, \qquad i = 1, 2$$

It is plotted in Fig. 3b. Along the trajectories we have $r_1 \neq r_2$ except for some discrete points. By Theorem 1, all state variables of both satellites are observable in time intervals that do not contain the isolated time moments of $r_1 = r_2$. However, from the uniqueness of solutions of differential equations, we can conclude that all state variables are observable on any time interval, including the isolated points where $r_1 = r_2$.

In the following, we test the UKF filter using relative measurement

$$h(\cdot) = [\,x_1 - x_2 \quad y_1 - y_2 \quad z_1 - z_2\,]^T$$

The tuning parameters in the UKF are selected as follows:

$$Q = 0.01 \times I_{12}, \qquad R = 10^{-11} \times I_3, \qquad d\tau = 0.1$$

where I_k denotes the identity matrix with dimension k; $d\tau = 0.1$ is the sampling period in the scaled time unit, which corresponds to 135.2367 s in the real physical time unit. This is a long sampling interval, which makes the estimation a challenging problem. The initial guess used by the filter is shown in Table 3. It is about 20% off the real initial conditions.

In the first set of simulations, we assume no measurement noise. The convergence of the UKF estimation is demonstrated in Fig. 4. It can be observed that after $\tau = 4$ (or $t \approx 90$ min) the estimation converges. Denote the position estimation error as

Table 3 Initial guess used for the simulations

	Satellite 1	Satellite 2
$(\hat{x}, \hat{y}, \hat{z})$	(1.2, 0, 0)	(1.1867, 0, 0)
$(\hat{v}_x, \hat{v}_y, \hat{v}_z)$	(0, 1.2586, 0)	(0, 1.2714, 0.0004)

$$e_p^2 = (x_1 - \hat{x}_1)^2 + (y_1 - \hat{y}_1)^2 + (z_1 - \hat{z}_1)^2 + (x_2 - \hat{x}_2)^2 + (y_2 - \hat{y}_2)^2 + (z_2 - \hat{z}_2)^2$$

and the velocity estimation error as

$$e_v^2 = (v_{x1} - \hat{v}_{x1})^2 + (v_{y1} - \hat{v}_{y1})^2 + (v_{z1} - \hat{v}_{z1})^2 + (v_{x2} - \hat{v}_{x2})^2 + (v_{y2} - \hat{v}_{y2})^2 + (v_{z2} - \hat{v}_{z2})^2$$

The convergence property of the UKF is also demonstrated in the logarithmically scaled coordinate in Fig. 5. The stable position estimation error is within 0.1 km. Relative to the radius, the range of the position error is within 10^{-5}%. The stable velocity estimation error is within 10^{-4} km/s.

Remark 1: The simulations are based on sampled data measurements. The estimation error can be reduced by increasing the sampling frequency. For instance, if we choose the sampling period to be $d\tau = 0.01$ (13.5 s), the stable position estimation error is reduced to approximately 0.01 km and the stable velocity estimation error is reduced to 10^{-5} km/s (see Fig. 6).

In the next simulation, we assume that the output is corrupted by measurement noise, that is

$$h(\cdot) = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

where the noise (d_x, d_y, d_z) has normal distribution with zero mean and standard deviation $\sigma = 50$ m. The UKF parameters are chosen as

$$Q = 0.01 \times I_{12}, \qquad R = 10^{-11} \times I_3, \qquad d\tau = 0.01$$

The simulation results are shown in Fig. 7. The stable position estimation error is about 0.35 km and the stable velocity estimation error is less than 0.2 m/s.

B. Observability of Networked Multisatellite Systems

Assume a system of five satellites flying in orbits with radius between 8500 and 11,000 km. We assume that the satellites receive no information about their positions, velocities, and orbit elements from either GPS or ground control centers. Sat-1, sat-3, and sat-5 are equipped with sensors that can measure relative position within a limited range. Sat-2 and sat-4 are not capable of measuring relative

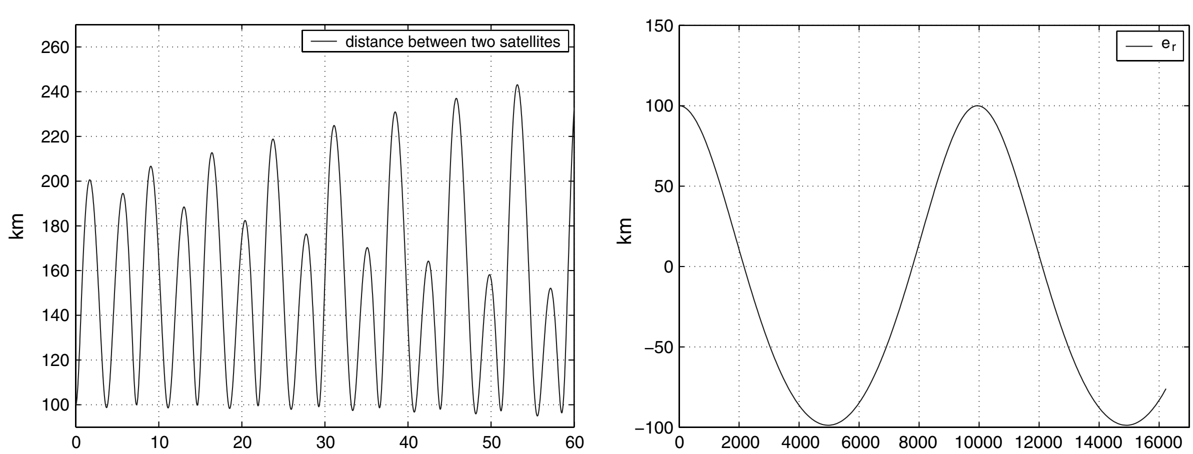

Fig. 3 The distance between the satellites and the radial difference.

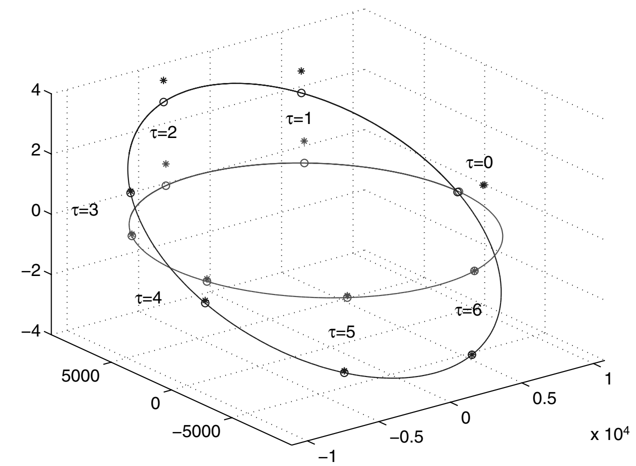

Fig. 4 Convergence of the observer. Circles denote the position of the satellite at different time instants, and asterisks denote the estimated position.

positions. We assume that the communication channels between the satellites suffer a 30% probability of interruption and failure with a uniform distribution, that is, at a time moment, the connectivity of a communication channel between satellites is a random variable with 30% probability of being zero (disconnected) and 70% probability of being one (connected). In this system, neither the sensor topology nor the communication topology is perfect. However, the conclusions in the following simulations are based on the assumption that the connectivity of sensor and communication topologies can last long enough for the filter to converge. The question to be answered is the following: is sat-2 observable to sat-1?

We assume that the effective range of the position sensors is limited by 200 km. The relative distances between the satellites are anywhere between 50 and 300 km. Figure 8 shows the relative distances in one period. Because of the large distance between sat-1 and sat-2, sat-2 is not directly observable to sat-1 in a large part of the orbit. For example, the matrix S of the sensor topology at $t = 1475$ is

$$S = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Obviously, both sat-2 and sat-3 are not directly observable to sat-1 at this time because they fly outside the limit of sensor range. Meanwhile, the communication graph is incomplete either because of the 30% probability of interruptions. For instance, the communication matrix at $t = 8028$ is

$$C = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The communication between three pairs of satellites are interrupted, $\text{sat}-1 \sim \text{sat}-2$, $\text{sat}-1 \sim \text{sat}-5$, and $\text{sat}-4 \sim \text{sat}-5$. In the simulation, both matrices S and C are functions of t depending on the relative distance and the probability distribution. As a result, the observability of the networked satellite constellation is highly dynamic. In this case, Proposition 3 provides an efficient

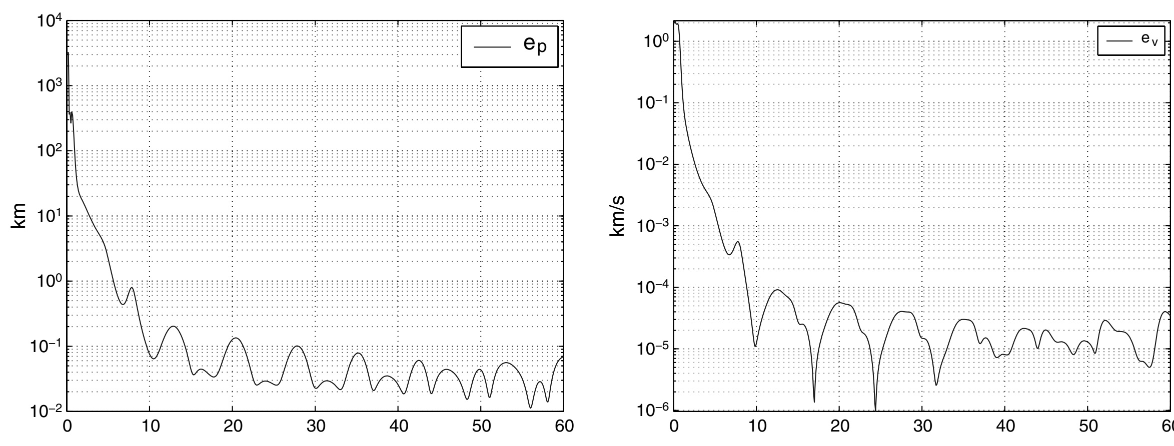

Fig. 5 Position estimation error and velocity estimation error ($d\tau = 0.1$).

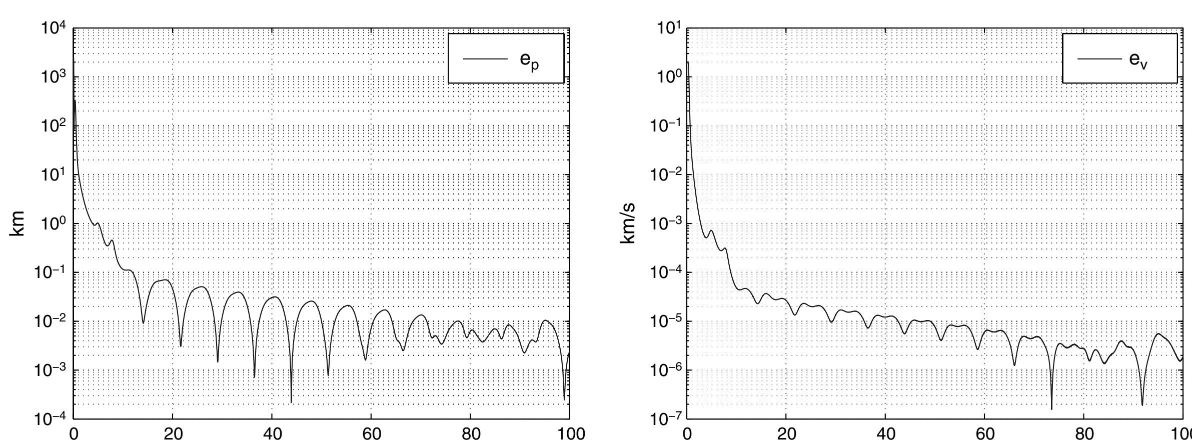

Fig. 6 Position estimation error and velocity estimation error ($d\tau = 0.01$).

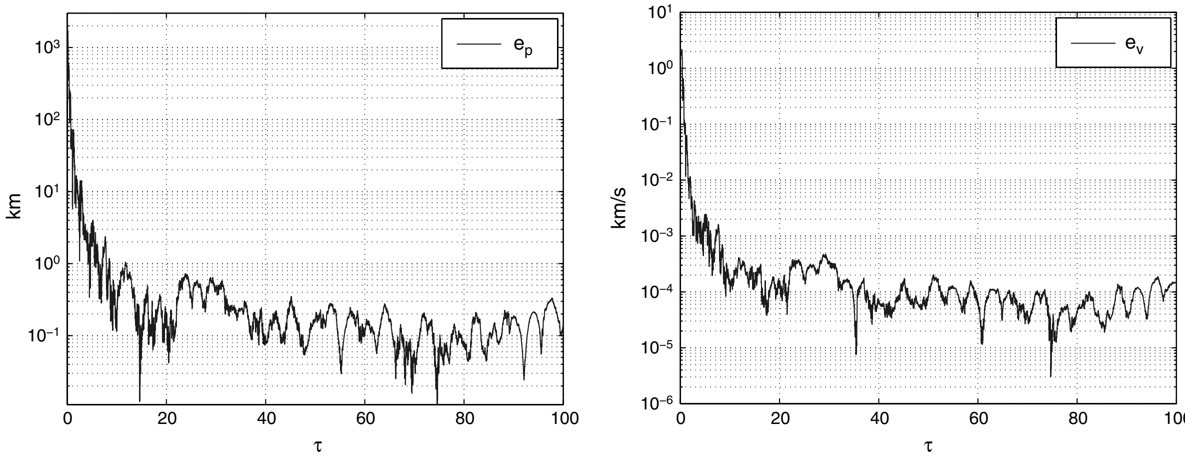

Fig. 7 Position estimation error and velocity estimation error under the measurement noise ($d\tau = 0.01$).

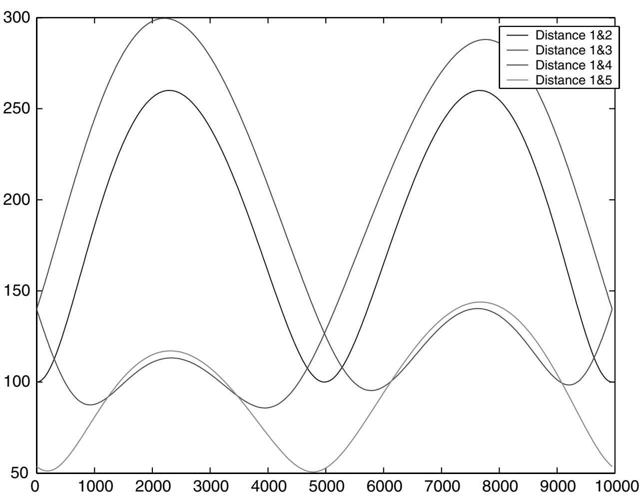

Fig. 8 Relative distance between sat-1 and other satellites in constellation.

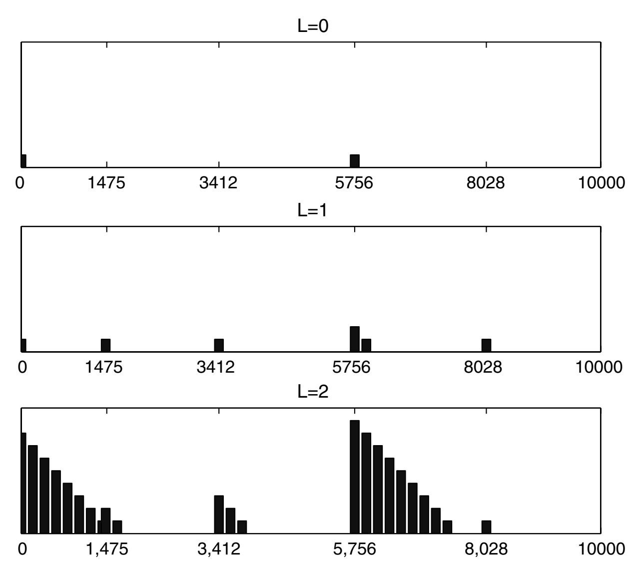

Fig. 9 Observability of sat-2 to sat-1 in a networked constellation. Horizontal axis: five time instants at which the observability is computed. Vertical bars: each bar represents a communication route of length *L*.

computational method of determining the observability using the combined information of sensor and communication topologies.

In the simulation, the observability of sat-2 to sat-1 is computed at five time moments $t_1 = 0$, $t_2 = 1475$, $t_3 = 3412$, $t_4 = 5756$, and $t_5 = 8028$. Because sat-2 is not always directly observable to sat-1, sat-1 must periodically request the state estimation from other satellites through communication. Depending on the length of communication routes (number of satellites used in the communication to pass information to sat-1), sat-1 may have multiple communication routes to acquire the state information of sat-2. In Fig. 9, each communication route is marked by a bar. In the first plot of Fig. 9, the length of communication is $L = 0$, that is, sat-1 measures and estimates the state of sat-2 directly. The two bars in this plot imply that sat-2 is directly observable to sat-1 at $t_1 = 0$ and $t_4 = 5756$. Sat-2 is not directly observable at t_2, t_3, and t_5. However, with the assistance of communication routes of length one ($L = 1$), the second plot in Fig. 9 shows that sat-2 is observable to sat-1 at all five time moments. At $t_4 = 5756$, there exist two different communication routes of length one through which sat-1 can acquire the measurement and state estimation of sat-2. If sat-1 uses communication routes of length three, then the third plot shows that sat-1 has many different routes to acquire the information, eight routes at t_1, two routes at t_2, three at t_3, nine at t_4, and one at t_5.

VI. Conclusions

It is proved that the orbits, the absolute positions, and the velocities of multiple satellites are observable using the measurement of relative positions only. This observability is verified by using UKF and numerical simulations. For networked satellite constellations, a system with incomplete communication and sensor topologies could be completely observable if the two topologies work in harmony with each other. This observability of networked systems can be computed using matrix algebra. These results demonstrate the possibility of significantly reducing the dependency of multisatellite operations on the support of ground control centers and GPS. The proved observability is valuable for the development of intelligent and autonomous space systems and for missions to deep space where GPS signals are not available.

The observability in Theorem 3 depends on the distance between satellites. In general, the observability becomes more sensitive to noise if the relative distance is small. The measure of observability as a function of relative distance is a topic of current and future research.

References

[1] Doolittle, C. M., Chavez, F. R., and Lovell, T. A., "Relative Orbit Element Estimation for Satellite Navigation," *AIAA Guidance, Navigation, and Control Conference*, AIAA Paper 2005-6462, 2005.
[2] Woffinden, D. C., and Geller, D. K., "Relative Angles-Only Navigation

and Pose Estimation for Autonomous Orbital Rendezvous," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 5, 2007, pp. 1455–1469.
doi:10.2514/1.28216

[3] Li, X., and Goodzeit, N., "Observability of a Geosynchronous Spacecraft Attitude Determination System," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 6, 2003, pp. 972–978.
doi:10.2514/2.6925

[4] Mukundan, R., Narayanan, R. R., and Philip, N., "A Vision Based Attitude and Position Estimation Algorithm for Rendezvous and Docking," *Journal of Spacecraft Technology*, Vol. 4, No. 2, July 1994, p. 6066.

[5] Yim, J. R., Crassidis, J. L., and Junkins, J. L., "Autonomous Orbit Navigation of Two Spacecraft System Using Relative Line of Sight Vector Measurements," American Astronautical Society, Paper 04-257, 2004.

[6] Alfriend, K. T., and Yan, H., "An Evaluation and Comparison of Relative Motion Theories," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, March–April-2005, pp. 254–263.
doi:10.2514/1.6691

[7] Gauthier, J-P, and Kupka, I. A. K., *Deterministic Observation Theory & Applications*, Cambridge Univ. Press, Cambridge, England, 2001.

[8] Kang, W., "Moving Horizon Numerical Observers of Nonlinear Control Systems," *IEEE Transactions on Automatic Control*, Vol. 51, No. 2, 2006, pp. 344–350.
doi:10.1109/TAC.2005.863509

[9] Gong, Q., Ross, M. I., and Kang, W., "A Pseudospectral Observer for Nonlinear Systems," *Journal on Discrete and Continuous Dynamical Systems: Series B*, Vol. 8, No. 3, 2007, pp. 589–611.

[10] Kang, W., and Barbot, J.-P., "Discussions on Observability and Invertibility," *Proceedings of IFAC NOLCOS*, Elsevier, Amsterdam, Aug. 2007.

[11] West, D., *Introduction to Graph Theory*, Prentice–Hall, Upper Saddle River, NJ, 1996.

[12] Jungnickel, D., *Graphs, Networks and Algorithms (Algorithms and Computation in Mathematics)*, Springer–Verlag, Berlin, 2008.

[13] Julier, S. J., and Uhlmann, J. K., "Unscented Filtering and Nonlinear Estimation," *Proceedings of the IEEE*, Vol. 92, No. 3, 2004, pp. 401–422.
doi:10.1109/JPROC.2003.823141